# Continuous simulation of variations during the design of endless fiber reinforced composite structure assemblies

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## Abstract

Using composite materials like endless fiber reinforced plastics offers numerous advantages. Nevertheless, the material properties of composite materials lead to new challenges in the manufacturing and assembly process. Additionally, variations and uncertainties occurring in the different production steps lead to increased production costs. Therefore, appropriate consideration of variations is needed and can be achieved through a continuous analysis of variations and their effects on the parts‘ quality using methods of tolerance management. The contribution proposes the development of such a simulation strategy.

## Keywords

*Fiber reinforced plastics, Tolerance management, Tolerance-cost optimization, Manufacturing simulation, Assembly simulation*

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## 1. Introduction and motivation

Using composite materials like endless fiber reinforced plastics (FRP) offers numerous advantages, like a high lightweight potential due to low density and high strength and stiffness [1]. Nevertheless, the material properties of composite materials lead to new challenges in the manufacturing and assembly process. Since composite parts consist of multiple layers of anisotropic fibers, the number of design parameters is significantly higher in comparison to conventional materials. Variations and uncertainties occurring in the different production steps from design to manufacturing and assembly lead to increased costs, quality loss, and in the worst case, scrap parts. Therefore, an appropriate consideration of variations is needed and can be achieved through continuous analysis and tolerance management to define tolerance ranges within which the design parameters must lie.

## 2. Related work and open research questions

Manifold variations along the various process steps needed to produce FRP assemblies must be considered. Several research works focus on analyzing uncertainties in one sub-process or optimizing one of the process steps.

The influence of laminate parameter [2, 3] and process parameter [4] variations on individual FRP components has been well studied. The analysis of variations and their influence on FRP assemblies is currently a subject of research. The influences of process variations in the curing of an FRP assembly are investigated in [5]. An analysis of the stresses that occur during the joining process is presented in [6]. In [7] the importance of calculating residual stresses and geometric deformations from the joining process of variated components is highlighted. The mentioned research works focus on single sub-processes, although Polini and Corrado emphasize the significance of the continuous consideration of variations in [8]. Therefore, they present an approach for estimating the geometric variations within an FRP assembly using the ‘Method of influence coefficients’ (MIC), but only consider fiber angle variations [9].

Nevertheless, there is still an open research question: How can the whole process from a single FRP part to a composite structure assembly be continuously simulated, considering variations in all steps?

To answer the question posed, the production process is split into sub-processes and the source of variations in each step is investigated (Sect. 3). In Sect. 4, a continuous method for simulating the sub-processes with variations is proposed. Sect. 5 explains a way to deal with the variations and Sect. 6 gives a brief overview of ways to reduce the computational effort.

## 3. Variations in the product development process of composites

Variations occur in multiple steps of the product development process of FRP assemblies. An overview of these steps is given in Figure 1.

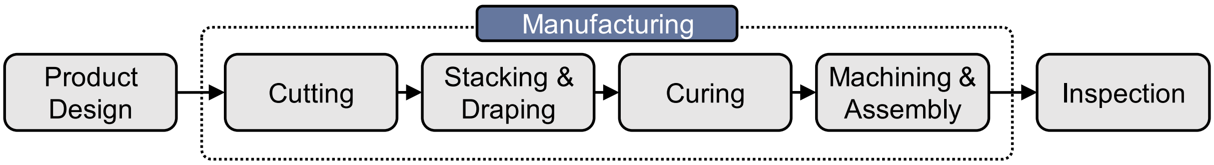

Figure 1: Product development process of FRP assemblies

The first step is the manufacturing of a single part. A commonly used technology for manufacturing FRP parts is the prepreg technology. A prepreg consists of ‘pre-impregnated’ fibers and the matrix, e.g., epoxy resin [1]. That way, manual impregnating is not needed and therefore better quality and higher fiber volume fractions can be achieved. The prepreg can be the first source of variations as the material properties like fiber volume fraction or thickness can vary [10]. The prepreg tapes are **cut** and placed in rows next to each other in a mold, either by hand or by automated tape laying (ATL) [5]. By **stacking** multiple plies on top of each other in a specified direction, the whole laminate is manufactured as designed beforehand. Possible variations are a wrong order of the layup sequence, missing plies and most importantly variations of the ply angles, which lead to different directions of the fibers [10]. The fiber angles of the plies are an important design parameter because they significantly influence the strength and stiffness of the FRP part due to the highly anisotropic material properties. Additionally, local reinforcement patches are used to increase the strength and stiffness of highly stressed areas while keeping the lightweight design [11]. The optimal location, shape and direction of the reinforcement patches can be calculated by laminate optimization approaches [12]. On the downside, the reinforcement patches' positions, sizes and orientations also underly variations resulting in lower reinforcement than expected.

Another challenge is the **draping** of the laminate of doubly curved parts, as it can cause significant shear deformation and wrinkling [13]. Thus, especially the variations of reinforcement patches in doubly curved areas can have a huge impact when they are not placed ideally as variations may accumulate. The consequence of variations in the laminating and draping process can be a lower quality of the part, which results in deformations, lower strength and, in the worst case, premature failure.

After draping all plies and patches, the next step in which variations occur is the **curing** process. A fluorinated ethylene propylene (FEP) film or a release agent is used to facilitate the demolding after the curing process by preventing the sticking of the laminate to the tool. Then a vacuum bag is applied and the whole stackup is sealed with sealant tape [14]. Subsequently, an autoclave is used, which allows the application of pressure on the laminate during the curing process [1]. After the preprocessing, the parts are cured according to the manufacturer's recommended cure cycle (MRCC). Figure 2 exemplarily shows such a cure cycle, which always consists of 2 heating phases, 2 dwelling phases and a cooling phase. Firstly, the temperature is increased from room temperature to 120 °C at a rate of 2 °C/min. In the second stage, the temperature is held at 120 °C for 60 min. Then the part is heated up to 180 °C and held at this temperature for another 60 min reaching the rubbery state. Finally, the part is left to cool down and reach the final glassy phase before removing it from the mold. [15]

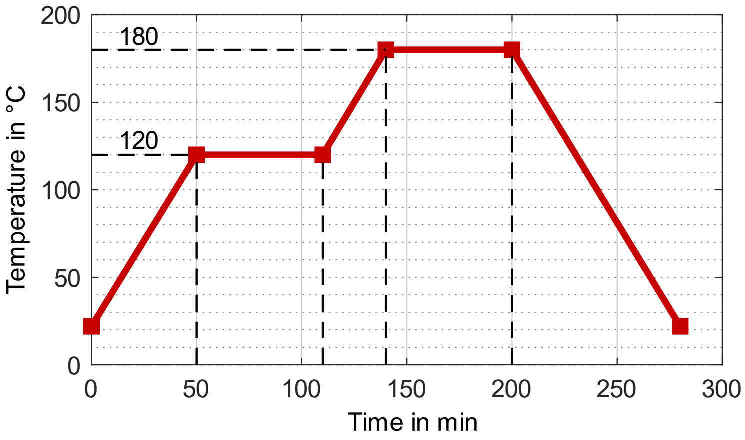

Figure 2: Exemplary cure cycle for carbon fiber reinforced plastics [15]

While curing the composite part, the anisotropic thermal and mechanical properties cause residual stresses and geometrical deformations [5]. The reason for the residual stresses is the difference in the coefficients of thermal expansion (CTE) between fibers and resin. Although this does not cause distortions on the macro scale, it leads to thermal anisotropy [16]. Since the CTE of the fibers is smaller than the CTE of the resin, the part expands or contracts more in the resin-dominated direction, which can lead to cracking and failure [17]. The mismatch between the in-plane CTE of the laminate and the through-thickness one and the chemical shrinkage of the resin causes the reduction of enclosed angles, the spring-in effect [18]. Besides spring-in, the second type of deformation that occurs during the curing process is warpage of flat sections. Warpage is, contrary to spring-in, depending on extrinsic process parameters like tool material, bagging arrangement [19], tool-surface roughness, the prepreg material [20] and the length-to-thickness ratio of the laminate [21]. Another major reason for warpage is an asymmetrical stacking sequence and should therefore be avoided [22].

As pointed out, the cure cycle plays an important role in the FRP manufacturing process and variations of the recommended cure cycle therefore have a huge impact on the final part geometry.

Finally, variations also occur in the **machining** and **assembly** process of FRP parts. After the positioning of the parts, they need to be held in place by clamps or bolts [9]. Because of the often significant deformations after curing, this can again lead to stresses inside the parts of the assembly. The fastening can be another source of uncertainties, as the holes for bolts or the positions of the clamps can be subject to variations. Additionally, the fastening sequence may have an influence on the resulting stress state and geometry of the assembly [23]. FRP parts are mostly joined by glueing. After positioning, fastening and glueing, the clamps or bolts are removed and the assembly eventually springs back due to residual stresses. This leads to geometrical deformations of the assembly. These resulting deformations can be computed, but are subject to variations in the assembly and manufacturing process.

The last section briefly describes the various variations that can occur in the product development process of FRP assemblies. In the following section, a strategy for simulating the process steps, including the consideration of variations, is presented.

## 4. Simulation of the manufacturing process of FRP assemblies

Figure 3 gives an overview of the developed simulation strategy. In the following, the substeps are described in detail.

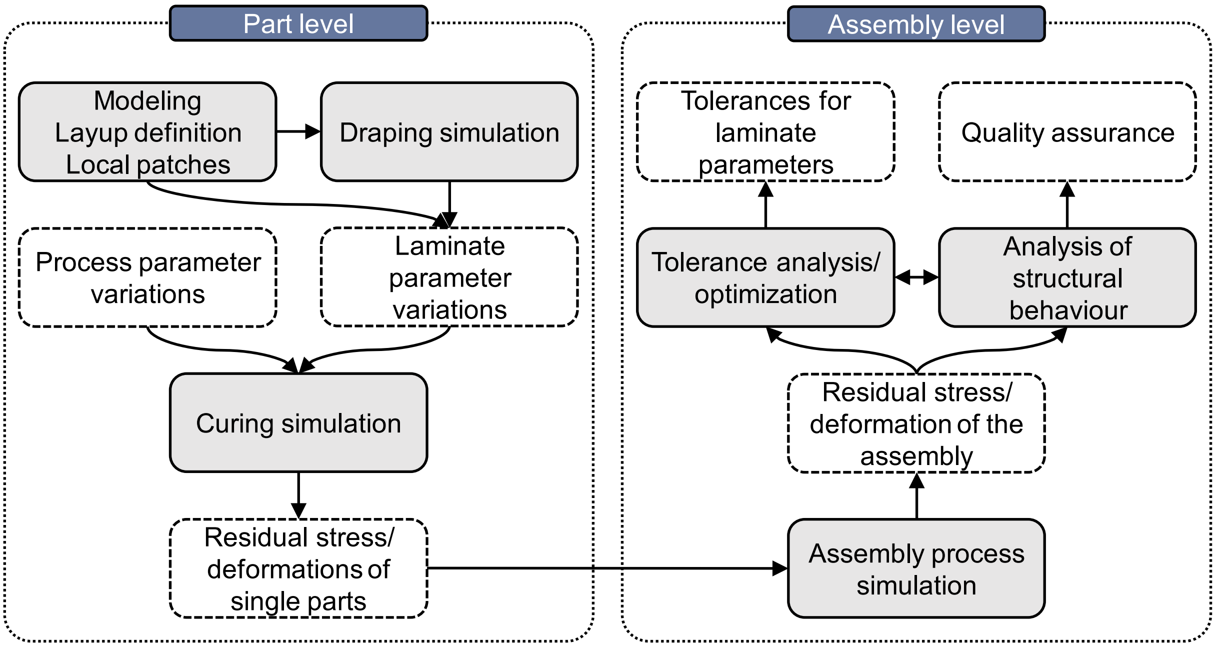

Figure 3: Overview of the developed simulation strategy, based on [24]

The first step of the simulation strategy is the design and **modeling** of the parts‘ geometry in CAD. Since composite parts are mostly thin-walled structures, they should be modeled as shell geometry. The shell represents the middle layer or the tool surface and the thickness is later calculated from the material properties and layup information. The next steps are creating a finite element mesh, modeling the stackup and defining the parameters of the laminate, e.g., the number of plies and the fiber orientation of the individual plies. When prepregs are used, material properties like thickness, fiber volume fraction and CTE depend on the manufacturer. Variations from the defined nominal values of the laminate parameters can be simulated by adding random variations using sampling methods like Latin Hypercube Sampling (LHS). Finally, the position of local reinforcement patches and their sizes and orientations have to be defined. This information can be obtained from topology and laminate optimization in the design phase.

The next step is a **draping simulation** to respect the fiber angle variations in curved and especially in doubly curved areas due to draping of the laminate and patches [13]. It is described in detail in the following section, since previous research works do not yet consider draping variations when simulating the whole production process. Draping simulations can be divided into mechanical approaches and kinematic methods [25]. For the mechanical draping, the fabric is assumed to be a solid continuum with anisotropic, elastic properties and friction. A finite element analysis (FEA) is performed to simulate the deformations due to draping. Since this approach is computationally expensive, kinematic draping simulations are more popular. They are based on the assumption that the undeformed fabric consists of perpendicular, interlocked and inextensible fibers [25]. Kinematic draping algorithms are based on four steps [26]:

1. Pick a point $\alpha_0$ on the mesh as starting point for the draping algorithm
2. Draw four orthogonal geodesic lines starting from $\alpha_0$
3. Each of the lines is split by placing equidistant points on the lines (alpha points)
4. Fill the grid with points by using an iterative scheme (beta points)

Kussmaul et al. developed a novel kinematic draping algorithm based on the work of Van der Weeën [27] and Tucker [28] using conformal mapping [26]. After picking the starting point for the draping algorithm, the 3D submesh is selected and conformally mapped to a 2D mesh. The draping is then performed in 2D and the results are mapped back to the 3D mesh. This approach greatly simplifies the draping algorithm and leads to the real fiber angles for every mesh element after draping. Even though new uncertainties due to mapping errors may arise, the kinematic draping allows integrating random variations like variations of the starting point location on the mesh and variations of the patch size and orientation. The impact of patch parameter variations on the structural strength of FRP parts and assemblies has to be investigated in future research.

Figure 4 shows an exemplary application of the draping simulation for a rectangular patch over a doubly curved surface. The draping simulation is based on the work of Kussmaul et al. [26] and was extended by adding parameter variations. Variations that are taken into account are patch location, patch orientation and patch size. In Figure 4a, the 3D submesh with the calculated alpha points is shown as well as the nominal draping starting point and the nominal draping direction. The length of the geodesic lines is given by the variated length and width of the patch. The position of the draping starting point as well as the patch orientation are also deviated from their nominal values as illustrated in Figure 4. The nominal patch angle is 0°, whereas the real patch angle is 4.5°. Figure 4b shows the submesh with the alpha points mapped to 2D using conformal mapping [29]. The beta points are then calculated and for every mesh element, whose center point is enclosed by beta points, the resulting fiber angles are computed. In this example, they lay between 2.19° and 8.21°, which means the draping caused a fiber variation between -2.31° and +3.71°. Depending on the size and position of the patch, different mesh elements are chosen and therefore have an impact on the simulation results. The final step is the backmapping of the fiber angles to the 3D mesh. The result is a list of elements that are part of the particular reinforcement patch and a fiber angle for each of these elements.

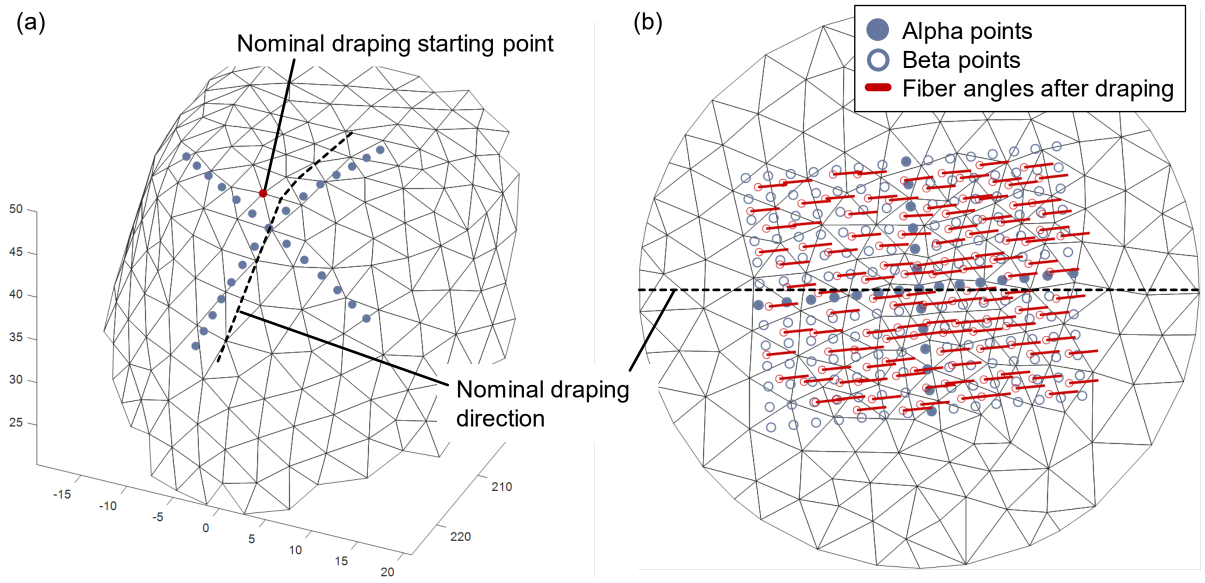

Figure 4: (a) 3D submesh with alpha points, (b) 2D submesh with draped fiber angles

The draping simulation and the sampling of variations of the design parameters are performed in MATLAB and integrated into the FEA by modifying the solver input file.

After modeling and draping the next process step is the **curing simulation**. As explained in the last section, the curing is a complicated process where variations can have a major influence on the resulting geometry. Curing simulation can be performed in different levels of detail. For the most basic simulation, isothermal conditions are assumed and only the cooling is simulated. A 3-2-1 support is used to prevent rigid body motion while allowing the part to deform during cooling. For a more precise curing simulation, the ANSYS Composite Cure Simulation (ACCS) software has to be used. It allows to simulate the thermal-chemical reaction and to predict the development of residual stresses and distortions during the curing of composite parts. The cure cycle has to be defined before starting the curing simulation. Especially when curing thick laminates, there may be a temperature gradient through the thickness of the part as a result of different temperatures on the top and bottom of the laminate. With a transient thermal analysis, it is possible to calculate the development of cure and the temperature profile and then use the information on distortions and residual stresses in a structural analysis, e.g., to simulate a real use case with an applied load.

Finally, the **assembly process** of FRP parts needs to be simulated. In a first step, the parts that are deformed due to the curing process are imported and positioned in the FE software. The clamping is simulated by adding boundary conditions that reverse the deformation of the parts at the joining points. This again leads to residual stresses. Joining is simulated by adding a bonded contact between the two parts after forcing the parts into their nominal shape. After the bonded contact is established, the boundary conditions at the joining points are removed and the now joined assembly is released. Due to the residual stresses the assembly springs back and then reaches its final state. Subsequently, an **analysis of the structural behavior** can be done, and quality loss or malfunction due to variations in the production process can be predicted. A **tolerance analysis** allows to identify contributing tolerances that can be modified by applying a **tolerance-cost optimization** to achieve higher quality, manufacturability and assemblability.

## 5. Dealing with variations during the manufacturing process

In the last section, it was explained how the manufacturing process of FRP assemblies can be simulated and how variations of design parameters can be integrated. The question that arises is how to deal with these variations? Since variations are unavoidable and ubiquitous, tolerances are used to define a range of permissible variations. Methods, models and tools of the tolerance management are used to define optimal tolerances that guarantee the functionality despite the occurrence of geometrical variations. A promising approach for single FRP parts without considering draping effects was developed by Franz et al. [10] using tolerance-cost optimization [30]. Figure 5 provides an overview of the tolerance-cost optimization workflow.

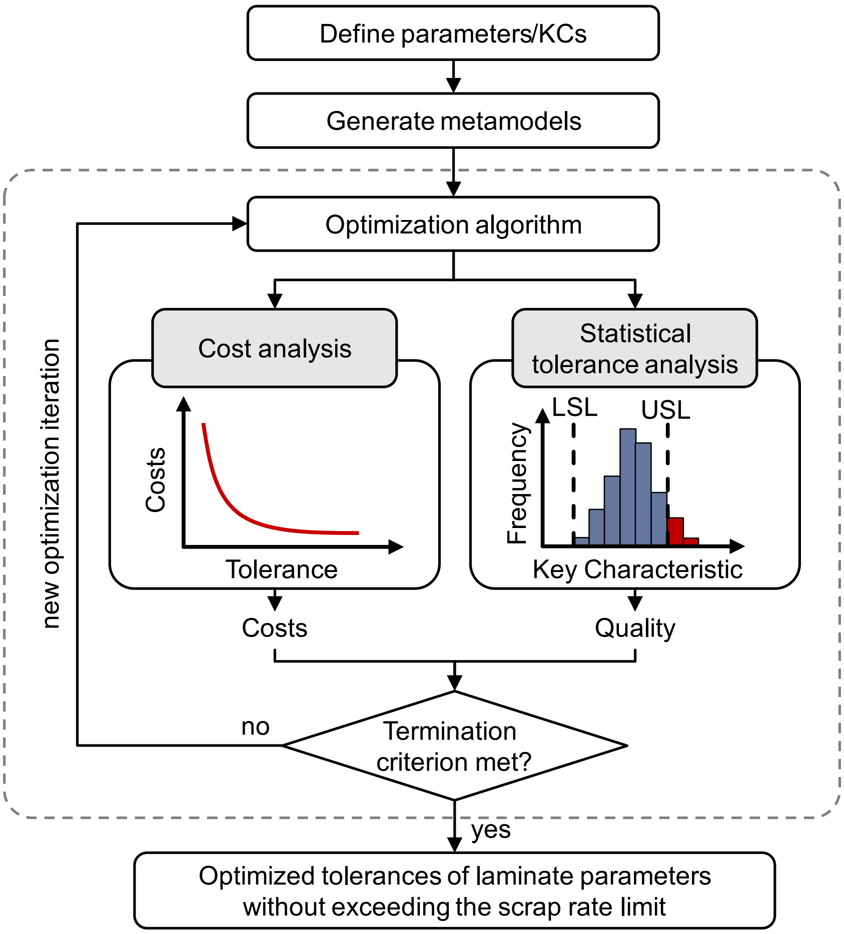

Figure 5: Flow chart of the tolerance-cost optimization, based on [10, 30]

The optimization goal is to find tolerances that are as wide as possible to keep the costs low but as narrow as needed to keep the scrap rate below the specified limit. The first step is the definition of key characteristics and limits for composite material failure criteria, e.g., Tsai-Wu failure criterion [31] and Puck failure criterion [32]. The optimization consists of two main components, an objective function which is minimized and a constraint function which has to be fulfilled. The objective is the minimization of the total costs and is calculated in the cost analysis using tolerance-cost curves. The constraint function applies a statistical tolerance analysis which measures the total scrap rate for the given tolerance values. Because statistical tolerance analysis uses a large number of samples to calculate the resulting scrap rate, it can quickly become time-consuming. [30] In the case of FRP parts and assemblies, the scrap rate cannot be formulated as a simple function, but has to be calculated by an FEA for every sample. After costs and scrap rate are calculated, the termination criterion indicates whether the optimization is completed or a new optimization iteration is started, which is repeated until the termination criterion is met. Metaheuristic optimization algorithms are best suited in this case because of the non-linearity and non-continuity of the constraint function as well as the possibility of several local minima of the objective function [30].

In contrast to the classical tolerance-cost optimization, which is used to optimize geometrical tolerance values, in this case it is used to define tolerance values for the laminate parameters, for reinforcement patch parameters and/or for curing and assembly process parameters. Hence, close cooperation between design and manufacturing is mandatory to achieve the best results. The goal is the definition of tolerances in a way that the functionality, quality and assemblability are given even when variations occur.

## 6. Ways to increase the computing efficiency

Due to the large number of samples that is needed for statistical tolerance analysis and the iterative optimization algorithm, a high number of simulations has to be performed. This can lead to excessive computation times, making it crucial to investigate ways to increase computational efficiency.

Surrogate models or metamodels can be used to reduce computing times by approximating the deformations and stress state. Figure 6 provides an overview of the whole metamodeling process, which is described in the following.

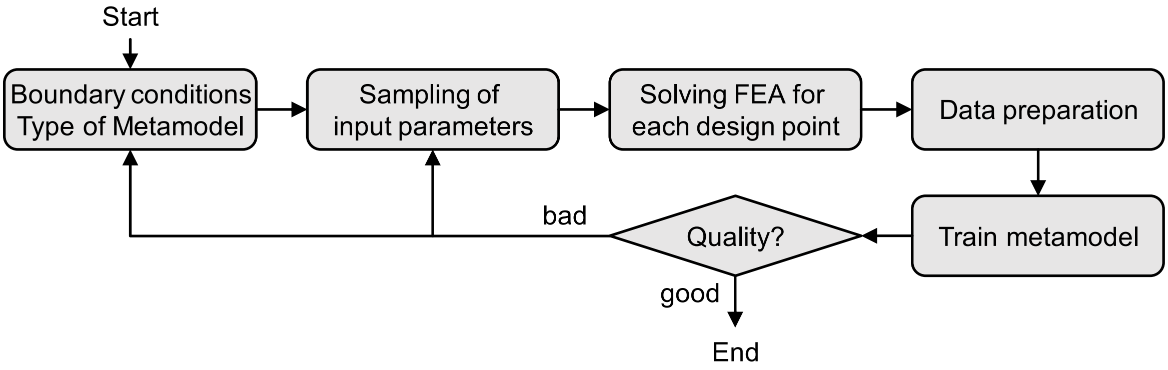

Figure 6: Flow chart of the metamodeling process

The first step of the metamodeling process after defining the boundary conditions and choosing a type of metamodel is data generation. A metamodel requires an input data set and corresponding output data. The input data can be data from experiments or, as in the present case, simulation data. Therefore, a sampling of input parameters is needed, e.g., by using LHS to generate variational parameters with a uniform distribution. The LHS is an efficient sampling method as it covers the sampling space uniformly and thus leads to a lower number of samples needed in comparison to random sampling. [33] This is useful because the output data, e.g., deformations, stresses and failure criteria, have to be calculated by solving an FEA for each design point. The output data then has to be matched with the input data and prepared for the metamodel training. After the data preparation, the metamodel has to be trained. The trained metamodel can now predict the results of any set of input parameters within the parameter boundaries without solving a time-consuming FEA. The quality of the metamodels can be evaluated, for example, by calculating the root mean squared error (RMSE) or the coefficient of prognosis (COP) [34]. If the quality is not good enough, another type of metamodel may be more suitable or more input data is needed. Otherwise, the metamodel can be used in the tolerance analysis that is needed for tolerance-cost optimization.

In addition to using metamodeling techniques, another way to reduce computation time is to make the FEA itself more efficient. A curing simulation with a transient thermal analysis takes significantly more time than one without the transient analysis, but is only necessary for very thick laminates. For relatively thin laminates, a uniform temperature distribution can be assumed. Additionally, since only the final state of cure is relevant for the continuous simulation, the time steps of the FEA can be reduced to further reduce the computing time.

## 7. Conclusions and future work

The proposed framework enables a continuous consideration of variations along the manufacturing process of endless fiber reinforced composite structure assemblies. Thus, material parameter variations as well as manufacturing and assembly process variations are taken into account by using specialised simulation methods. The methodical approach focuses on the investigation of FRP assemblies and the interaction and consequences of the variations from the individual substeps in an efficient way. In combination with tolerance optimization approaches, this ensures high product quality.

With a systematic investigation of different case studies, the performance and efficiency of the presented method will be determined in future research activities and recommendations for the tolerancing of FRP assemblies will be derived. Moreover, a comprehensive sensitivity analysis indicating the influence of each parameter variation allows further reduction or simplification of simulation models to speed up the design process for practical use.

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