4 DECISION-MAKING IN DESIGN: A COMPARATIVE STUDY

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This paper presents a comparative study of the design processes of final-year industrial design students of 2 countries while conducting an individual design task. This task was identical for both groups making this comparison possible even though the studies are 15 years apart. This new study gave us opportunity to observe new aspects initially not focused upon. The operational aims of this study are the identification and comparison of the way senior design students in both groups take decisions, the relation with design moves along the process, and the factors influencing the decisions and moves.

For that purpose both verbal protocol analysis studies (VPA) were analyzed on the basis of activities and decision-making moments described in terms of reasons behind it and goals intended to be achieved through it.

The results indicate the relevance of two aspects: a) the abductive reasoning that supports designing gains visibility through analysis based upon decision-making; where idea generation plays a key role. b) the notion of design as a decision-making process could bridge, in a meaningful way, design education and design practice in organizations.

Keywords: Design Process, Verbal Protocol Analysis, Decision-Making in Design, Information in Design.

1. INTRODUCTION

The use of verbal protocol analysis in design now has built up a tradition of about 20 years. The method, meant to get an understanding of the cognitive process, has proved to be efficient in describing a number of characteristics of the design process. Examples are the use and the role of drawing, the information-seeking behaviour, and the decision taking process. The results should be useful to support and improve problem solving in design practice by developing an appropriate methodology, and to train design students and practitioners in a more effective way.

As Longueville *et al.*¹ noticed in recent years a number of proposals have been introduced for the study of decision-making processes in knowledge areas such as management, cognition, engineering design, artificial intelligence etcetera. Those approaches are of different nature: prescriptive and descriptive. The first approach has been widely used to support decision by prescription, optimization and new decision-making process deployment". Our work integrates the second approach, the descriptive approach in the way Longueville *et at*¹ defines it: as an approach aiming "at modelling in order to study, understand, represent and re-use existing decision-making processes". In our opinion, the most relevant contribution lies in the analysis of the relationship between the decision-making process and the quality of the result. Reason is our belief that product development should solve a profit-maximization problem.² In controlled protocol studies one can only simulate part of this product development process, the conceptual stage of the product. But even within these constraints this process shows something of the product development organization in terms of a sequence of steps that transform customer requirements into a satisfactory product design; and of the information flow governed by one decision-maker who make both design decisions and development decisions under time and budget constraints. It is a decision production system.³

Most academic studies over the last decades, however, lack this perspective of understanding how detailed design decisions affect profitability. Take for example John Gero's FBS (function-behaviour-structure) model of designing, first presented in 1990 and developed with his collaborators of the Key Centre of Design Computing and Cognition at the University of Sydney.⁴ In this model, recently discussed in Design Studies,⁵ decision-making is not addressed directly but in a diffused complex way. In spite of the fact that the model is both prescriptive and descriptive and that the authors claim to be unique in its versatility — as opposed to the limitations of all the models used until now, such as those developed in the sequence of Delft Protocol Workshop ⁶ – it lacks the ability to make possible a 'satisfying' (in terms of usefulness for designers, companies and education) empirical analysis of how and why the decision-making process leads to a certain quality of the result.

In this study, the focus is on the conceptual design phase, a phase in which information processing and decision-making is very intensive as a consequence of the generation and evaluation of alternative ideas.⁷

By studying the decisions made during the process and the factors that influence those decisions we will get a more detailed view on the effectiveness of the decision making process in terms of quality of the end result.

The questions addressed by this study are:

The decision-making process:

- What are the characteristics of the decision-making process in design (framing-enabling-key)
- How is the process of decision-making related to the generation of ideas and the quality of the final result?
- · Relationship with 'design moves'
- The role of different types of decisions
- What factors influence the decision process (knowledge/expertise, external information, sketching)

2. A COMPARATIVE STUDY BETWEEN DELFT (D) AND LISBON (L) PROTOCOLS

The aim of this study is to identify the way senior design students take decisions, the relation with design moves along the process, and the factors influencing these decisions and moves. In order to look for differences in educational and cultural background final-year industrial design students of 2 countries were compared while conducting an individual design task. This task was identical for both groups making this comparison possible even though the studies are 15 years apart. This new study gave us opportunity to observe new aspects initially not focused upon.

2.1. Method

Subjects. Protocol study D was undertaken in 1992 and included both 2nd-year and final-year students from the Faculty of Industrial Design Engineering at Delft University of Technology.⁸ For the purpose of this study we only compared the work Delft developed by the 10 final-year students (3 female and 7 male). They were selected out of 75 students on the basis of their average marks for the design courses (at least a 7 out of 10). Protocol L was conducted in 2007/2008 and its subjects were 14 students (11 female and 3 male) from the last year of the Design course at Faculdade de Arquitectura da Universidade Técnica de Lisboa. From the class of 17 they volunteered in the project.

Both studies had the same assignment that proposed the creation of one or more concepts of an industrial object –a litter-disposal system in the train — that called for the integration of aspects such as ergonomics, construction, aesthetics and business. Each design student had to perform the task individually.

Subjects were requested to think aloud during the process of solving this design problem. Both experiments had a similar information system that was only presented to the subjects at their demand.

Information was separated by topics and presented in cards that were handed by the experimenter who was present in the room.

Prior to the experiment subjects from both groups made a 10 minutes preliminary test with the thinking-aloud method by trying to solve aloud a cryptarithmetic puzzle.⁹

After the experiment both groups had a debrief moment, an interview, that addressed their opinions about the experiment, the way they performed in it, etcetera.

All protocols were videotaped and transcribed.

Encoding. Data was then coded taking into account not only the information asked for and used but also the activities developed, time spent in each, reflections made and decisions taken.

All protocols where translated in English (so they could be evaluated by both authors) and coded according to an encoding system for decision-making.. The encoding system presents all types of decisions encountered in both protocols that were analyzed on the basis of the transcripts. Next, the codes were categorized in three decision types: framing, enabling and key decisions. *Framing decisions* classify the decisions made during the period when a designer mentally 'frames' the object; *key decisions* are those made on moments when the (preparation of the) product creation occurs; and *enabler decisions* signify mental object representation instants. The decisions were related with seven activity categories (asking, reading, looking, getting material, modeling, sketching and reflecting).

Concerning the analyses done in Delft protocols workshop $Cross^{10}$ highlights those that "reinforce the importance of a concept as marking a key point in the process" (p. 70). This key point is in our study what we refer to as a *key decision*.

Furthermore the analysis developed by Günter *et al.*¹¹ is also important to mention. Their analysis of the design process has three main stages: clarifying the task, searching for concepts and fixing the concept, the two first ones being covered by our *framing decisions* category and the last one corresponding to the *enabler decisions*.

In his analysis of the Delft protocols Cross¹⁰ also recognizes the occurrence of a bridging concept between problem and solution that "synthesizes and resolves a variety of goals and constraints; and it occurs during a 'review period' after earlier periods of more deliberately generating concepts and ideas" (p. 70). This review period in our study corresponds in some cases to the end phase of *framing decisions* or even to a period of time where in the protocol graphics *framing decisions* alternate with *enabler decisions*, mostly of reflecting nature ones as it is observable in Figures 3 and next ones.

In addition to this Cross ¹⁰ claims the 'appositional' nature of design reasoning that is characterized by the development of function and form in parallel rather than in series, being a neglected aspect in almost all design process models.

This is clearly observable in both protocols that display — as Cross¹⁰ mentions — it an "exploration and identification of the complex network of sub-problems in practice (that) is often pursue by considering possible sub-solutions. In practice, designing seems to proceed by oscillating between sub-solution and sub-problem areas, as well as by decomposing the problem and combining sub-solutions" (p. 78).

Within that perspective *key decisions*, according to our encoding system, are taken when bridging occurs among partial models of the problem and solution that have been constructed side-by-side. In the words of Cross¹⁰ it is a 'bridge' that recognizably embodies satisfactory relationships between problem and solution. "(...) the recognition of a proposed design concept as embodying both problem and solution together (...); it is neither one nor the other, but a combination which resolves both together and allows either to be focused upon" (p. 78–79).

For the purpose of this analysis we created a graphic's layout that allowed us to establish the precise moment of each decision along the process, its nature (that is described in the encoding system) and the way it contributes or not to the proposed solution that is related with a *key decision* (orange for the first one; red for the second one). *Idea generation* (purple color) also makes part of this graph that also allows a visual perception of the density of decision type and of the decision flow per activity.

Design quality assessment. In the D protocol study each judge individually rated each design on a number of attributes such as creativity and technical quality, using a 10-points scale. Judges were 6 experts in the field of industrial design. The same judging procedure was followed in the L protocol

study, but here the judges received the complete transcripts of the individual protocols including all sketches. Judges were 7 experts in the field of industrial design both from academic and industry fields.

Selected sample. In order to give a detailed illustration of the decision-making process and the types of decisions we will only present the analysis of the first hour of three protocols from each country. With the examples chosen we try to enhance the diversity and uniqueness we can find in design processes.

2.2. Results

"Poor Results" – Protocols D1 (male, average rating = 5.7) and L1 (male, average rating = 4.6) Protocols D1 and L1 were the ones that had the lowest average and median rating according to the jury. Their similar results have complete different processes behind.

However, they show a striking similarity by not succeeding in processing information and in idea generation. As we can observe in terms of decision-making (see Figure 1) subject D1 has an expressive density in columns related with asking and reading information that is not sustained by reflection on information. This indicates a lack of information processing and subsequent application.

Subject D1 is unable during this period to formulate a *key decision* displaying only *framing decisions* that gave no origin to idea generation. On the other hand in Protocol L1, Figure 2, we can observe that



Figure 1. Protocol D1 – first 60 minutes.

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Figure 2. Protocol L1 - first 60 minutes.

not much information was asked (and from the one asked the focus was on the train and its interior being the rest ignored); less decisions were taken, and there was a fixation to an idea that boosts a reasoning in a circular way It is also to consider the negative reaction both subjects had to the brief.

Subject's L1 immediate reaction was to propose two contrasting solutions, one that had severe implications with the train structure and layout and another one that was defended until the end that consisted in augmenting the capacity of existent bin by stretching it until the floor. It was clearly a strategy of opposing extreme solutions to benefit the one that was more realistic. At first sight subject L1 presents a quick idea generation followed by a period of sketching and reflecting activities. However, when analyzing the contents of those activities we come to the conclusion that he is fixated in circular reasoning as is from the fact that (1) his sketching is not meant to search for ideas but sticks to the same statements, and 2) the reflections made are a repetition of statements in favor of the option he made.

"Best Results" – Protocols D6 (male, average rating = 8.5) and **L2** (female, average rating = 7.1) Both higher rated protocols display an intense reflective dialogue – subject decides, reflects/evaluates upon decision, and decides again. Each activity is developed having the reflection mode as a master. It appears that the activity itself must have a complementary role in this analysis: not the reading by the



PROTOCOL D - SUBJECT 6 - FIRST 60 M INUTES

Figure 3. Protocol D6 - first 60 minutes.

subject is important, but what he is reading, the selection that he makes of it (decision) and the way that selection is consistently propagated along the process in order to contribute to the final solution.

Also important is the nature of those reflecting decisions that have a clear applied goal — most of the decisions are related with the idea generation process and its materialization in its multiple aspects from technical aspects to ergonomic or aesthetic ones. Subject D6 has a clearly defined period of brainstorming — an idea generation moment to expose a novel possible concept (33:50–38:30 min.) that serves not only the purpose of finding new paths but also as an evaluation moment to previous ideas, some of which partially integrated in the final solution.

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Figure 4. Protocol L2 — first 60 minutes.

On the other hand subject L2 displays a strategy of continuous monitoring. Tests of her ideas that occur as 'extensions' to previous ones through sketching and modeling where functional aspects pay a key role. The detailed comprehension of the object and its feasibility and easiness to use are central in the design.

3. HOW IS THE PROCESS OF DECISION-MAKING RELATED TO THE GENERATION OF IDEAS AND THE QUALITY OF THE FINAL RESULT?

Reflecting on the foregoing analysis of the decision making process and looking for the 'logic' behind the ideas generated we like to highlight on the basis of for example the L2 protocol (see Figure 4) that not only radical shifts of perspective characterize the generation and materialization of an idea in design processes. By means of manipulating the idea, exploring it deeply in an attempt to expose its self-potential and relational potential one can enter a 'creative leap' that in the words of Nigel Cross¹⁰"...might be no unexpected dislocation of the solution space itself, but merely a shift to a new part of the solution space, and the 'finding' there of an appropriate concept" (p.65). That is, according to Cross, what characterizes creative design as exploration rather then search.

Cross's idea of "creative design being the apposite proposal of a concept which embodies novel features for a new design product" (p.65) presents us the creative cognitive act in design similar to "building a bridge between problem requirements and solution proposal" (p.66).

Subject D6's protocol, where it is evident that each activity is preceded and followed by a reflection moment, formulated in operative terms, is a clear example of an 'undergoing creative construction' that involves problem and solution as the dynamic and interdependent parts of the 'engine' driving the process. What is observable from the analyses of D6's exercise is that his accurate and critical attitude towards the task 'under construction' made it possible for him to question problem and solution settings in an evolutionary interdependent way, entering a dynamic design practice that is recognized by Dorst & Cross¹² as being more "a matter of developing and refining together both the formulation of a problem and ideas for a solution, with constant iteration of analysis, synthesis and evaluation processes between the two notional design 'spaces' — problem space and solution space. In creative design, the designer is seeking to generate a matching problem-solution pair, through a 'co-evolution' of the problem and the solution" (p. 434).

4. RELATIONSHIP WITH DESIGN MOVES

The study conducted by Goldschmidt¹³ and approached by Cross¹⁰ identifies what she calls 'critical moves' i.e. "one which has a relatively high number of links to other statements that succeed it. In spite the fact that she does not identify the *key decision* moments her linkograph work clearly shows that there exists some statements that have a high number of 'fore-links' i.e. subsequent statements that build onto, or refer back to, those statements.

This path of related statements is also identified in our figures where decisions that contribute to the final solutions are marked; either they are technical *enabler decisions* or a reflecting enabler that reinforces or confirms a path or marks an inflection of direction.

In both protocols the most significant moves have do to with the decision to change the location of the litter disposal system. The new placement of the object determines the re-arrangement of all the constraints and variables of the problem and determines a change of paradigm that corresponds to a *key decision*. The pieces of information that contribute most to the need of finding a new place for the object are: images of the interior of the train that shows the actual location of the bin and that makes problems of capacity/dimensions evident to the designers, reach and interference with passengers' space/commodity and information about types of garbage — that especially in Protocol L lead to the idea of separating the garbage and thus finding a place that can support that feature.

5. WHAT FACTORS INFLUENCE THE DECISION PROCESS

There are several factors that influence the decision-making process. Among them we distinguish: a) the expertise/knowledge of the subject (that includes capitalized knowledge reuse¹ i.e. the reuse of any knowledge capitalised from the same project or other projects, b) the information content and the way subjects value it along the process; and c) sketching as a means of searching the solution space.

5.1. Expertise

Many studies have been conducted on expertise in diverse domains ranging from chess to physics and arts, and from novices to experts. The key aspects that define expertise seem to be: (1) quantitative and qualitative training, (2) motivation, and (3) acquiring complex mechanisms for controlling, executing and monitoring their performance. As the expert in the field of expertise, Ericsson¹⁴ claims: "The

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acquisition of reproducible superior performance on domain-specific tasks goes beyond accumulating knowledge. The development of high levels of skill requires the acquisition of representations that allow efficient control and execution of performance as well as mechanisms that support planning, reasoning and evaluation that mediate further improvement and maintenance of high levels of performance (p. 238). See also Cross' summary of expertise in design studies.¹⁰

In the Delft protocol study expertise was one of the research questions studied by comparing novices and final year design students. The most striking findings in this study were that the creativity of the solution was not dependent on the level of expertise, while the information-seeking behaviour definitely was.

In comparing the D and L protocols there are no differences in years of design education: both groups were in their 5th year. Any difference in expertise has to be attributed to either idiosyncratic factors or to differences in design education. The two design programmes differ indeed completely. While Delft aims at integrating engineering and design aspects in a business context, Lisbon is characterized by a humanistic and generalistic approach that does not emphasize technical and business aspects; Lisbon being more governed by the design studio. The absence of a consistent relationship with industry isolates design education in a progressive way; except for global industries in Portugal it is not so common for companies to adopt a designer in their development team, reason for a stalemate since a long time (but rapidly changing by a dramatic change in design programs). These differences are expressed in the protocols. Most striking is the difference in information-seeking behaviour, both in amount and in information type. L subjects are focused on information that they master, contrary to most D subjects who ask for more and more diverse information. Another difference is the assumption among L subjects that they themselves are prototypical for the average user; D subjects ask for information about user studies available.

Finally, in mechanisms of controlling, executing and monitoring their performances differences between D and L protocols are more difficult to tracer because here individual differences among all subjects are dominant.

5.2. Information Seeking, Selection and Focus

There are evident links between information requirement and decision-making. Information can open new paths of research for the solution but also serves the purpose of evaluation and/or confirmation of the existent hypothesis. That was visible in the case of information related with 'other solutions'. However it is important to notice that not all the information available was demanded and from the one required some was not used.

In the available information the one related with the images of the interior of the train, and with the current bin were the relevant ones, and they were asked for by all the subjects in both experiments. This type of information in almost all the cases concurred to explore alternative locations to the object that later boosted the generation of ideas, further developed in terms of shape and functional/constructive aspects.

As has been said it was observable that some of the information requested was ignored or not valued along the process and in the development of the solutions. That occurred in both D and L groups where information related with the producer and the railway company had a low (visible) impact on those that consulted it.

Also important to mention is the role of information created, the one that results from reflection either about demanded information or from retrieved information or even new one.

5.3. Sketching

Sketching influences decision-making in the way that it allows subjects to engage two types of reasoning as identified by Goldschmidt,¹⁵ one based on analogical or metaphorical thought, dealing with extracting new meaning from a sketch, that she describes as 'seeing as' and another type, the 'seeing that' deals with design consequences of this newly acquired meaning of the sketch. This role of

sketching as being "not merely an act of representation of a pre-formulated image (but) in the context (...) more often than not, a search for such an image" (p. 131) reinforces the importance sketching has in the decision-making process being evident the role of "reflection while sketching".

The exploration role sketching has was clearly observable in subject D9's protocol (Figure 6) that spent a considerable amount of time in it being conscious of its importance in idea generation. On the other hand there is the case of subject D6 (Figure 3) that stated his preference to make mental representations in searching and generating ideas being sketching used to refine the idea and communicate it effective ways.

Therefore it is also important to consider that as Van der Lugt¹⁶ states sketching affects the idea generation process (that is subject of an accurate scrutiny by decision-making process) in the way that: a) thinking sketches stimulates a re-interpretive cycle in the idea generation process (by means of its indeterminacy) b) talking sketches stimulates re-interpretation in the idea generation process; c) storing sketches stimulates the use of earlier ideas by enhancing their accessibility.

6. CONCLUSIONS

In general terms this study reinforced our awareness of the inadequacy of the existing methodologies that are mainly based on a logical kind of reasoning that differs from the abductive one in the way Peirce¹⁷ defined it a century ago. In fact this reasoning supports an activity similar to 'reverse engineering' i.e. "working backwards" as proposed by Polya¹⁸ that is defended by Peirce as being the only logical operation that introduces new ideas.

We also found out that the analyses provided by graphics based upon decision-making allow us to better catch the dialogue between problem and solution, envisioning the complexity of the process that is not covered when using activity based approaches.

In our future work we should take into account that designers conceive their activity as a problemsolving one which inhibits them to consciously think of it in terms of a decision-making one. That perspective, in our opinion, would bridge in a meaningful way, design education and design practice in organizations. In fact during most of the VP's process subjects are not aware that they are taking decisions. If they would realize, competing objectives when formulated in a conscious manner would probably steer the decision-making process towards the development of a balanced and effective solution.

Another finding was that design decisions (that are related to product form and manufacturing processes) are evidently more often listed in a conscious way than development decisions (that control the progress of design process).

Also evident was the fact that personal characteristics have a considerable impact on the decisionmaking process. Subjects with high self-esteem who are assertive and not averse of risk taking and uncertainty are more likely to decide in ways that allow processes to progress towards a consistent final solution. For those subjects decisions are not a process tool seen as a constraint but instead as an opportunity to proceed.

Regarding now the studied protocols in a more strict view there is a kind of primary "Pattern logic" approach to the problem that goes as follows: more passengers imply more garbage that implies a bigger bin or smaller ones in more quantity (capacity prevails as a criterion); This implied in almost all the cases the "reduction" of the solution to a bin instead of a system that was also influenced by the evaluation of the existent solution as well as by some information that integrated the specific company's information available.

The analysis of both protocol studies allows us to conclude that decisions that "made a difference" i.e. that implied *key decisions* and design moves, were almost always linked with: a) Location — that is linked with garbage volume (the most common subject's 'control constraint') and with passengers' use of the garbage and movements inside the train; b) Types of garbage — that influence dimensions and therefore location. Especially in L protocols there was a prevalence of the recycle concept even when ergonomics, usability, interface with users and employees and costs were assumed as being affected giving strength to the idea that "The principle overcomes the constraints".

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