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THE FOUNDATION FOR ROBUST DESIGN: ENABLING ROBUSTNESS THROUGH KINEMATIC DESIGN AND DESIGN CLARITY

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1. Introduction

Robust Design Methodologies (RDM) focus on developing engineering designs whose functional performance are insensitive to the geometric variation they are subjected to during production and use. Robust design literature offers comprehensive methods – quantitative as well as qualitative - for analysing and describing the robustness of a given design. Examples include Taguchi's Signal-to-Noise ratio [Wu 2000], differentiation of the so-called *transfer function* [Eifler 2011], Failure Modes and Effects Analysis (FMEA) [Bertsche 2008], and Fault Tree Analysis (FTA) [Bertsche 2008]. These methods are useful for analysing a given design and estimating failure rates and sensitivity to variations and hence take mitigating actions or optimise the design by adjusting the design parameters. However, it is often pointed out, that RDM lacks tools and methods for early-stage design and for synthesis of alternative solutions [e.g. Andersson1996]. The aim of this contribution is: 1) to describe why the principles of **kinematic design** and **design clarity** should be applied prior to other robust design activities, 2) to provide a step-by-step procedure that can be used during early-stage design to quantify the degree of adherence to these principles and 3) to highlight **ambiguity, abruptness** and other factors that affect the functional performance of a design. The principles described will be accompanied by a simple tool that can be used by engineering designers during early-stage (as well as detailed) design to quantify the clarity of a design. Finally, the principles and tools will be applied to two cases.

The research presented in this paper comes with over 30 years of combined experience of applying RDM in industry and therefore attempts to portray the industrial perspective. The paper therefore focuses on the dominant methods used in industry and some of the critical Design for Robustness issues.

2. State of the art: The correlation between robust and kinematic design

Taguchi is often refered to as one of the key players in robust design. Taguchi states, that there is a loss associated with *any* deviation of a performance characteristic (e.g. the force needed to push a button) from its target value and not just when performance lies outside the specified tolerance limits [Lochnar and Matar 1990]. In other words, an ideal robust design should have no variation in functional performance when a design parameter (e.g. the diameter of a hole) is varied. There are a finite number of sources of variation for design parameters. Although they are described and categorised differently in different literature, the sources are here described as:

- Production tolerances (e.g. due to variation in shrink percentage, process parameters etc.)
- Assembly tolerances (e.g. due to clearance around mounting screws)

- Load deformations (e.g. due to user loads, wind loads, gravity, etc)
- Variation due to ambient conditions (e.g. due to change in temperature, humidity, etc)
- Variation over time (e.g. due to wear, creep, swelling)

There is a wide variety of Robust Design Methodologies that describe how the influence of these variations can be reduced. They can be divided into categories depending on the information necessary to apply them, the output type (quantitative/qualitatitive), etc.. In [Eifler 2011], a classification of methods is presented. The typical methods focus on identifying and evaluating the design parameters either by experiments (Design of Experiments (DOE)) or analysis and hence using statistics to adjust the design parameters in order to improve the robustness of the design. However, in order to be able to conduct experiments, physical prototypes must be available and to set up an analytical (or numerical) parameter study, e.g. using computer simulations, calculations etc., the design must have reached a level of maturity where the design parameters have been identified and quantified. Furthermore, both the analytical and the experimental approaches are time-consuming, which presents a challenge during early-stage design, where the design changes so frequently that any analysis must be fast to conduct in order to be applicable. As a consequence, the robustness activities are not conducted until the design has reached a level where the conceptual solutions are somewhat frozen, even though the effects of *conceptual robustness* are greater than parameter optimisation. Despite this, there seems to be a lack of Robust Deisgn Methods for the design engineer to use during the early phases of product development. In Figure 1, the typical Robust Design Methods used in industry are placed according to the product development phase they are typically applied in, according to the authors' experience. It can be seen, that there is a lack of methods available for concept development and system-level design – the methods Kinematic Design and Unambiguous Design are new and will be presented later.

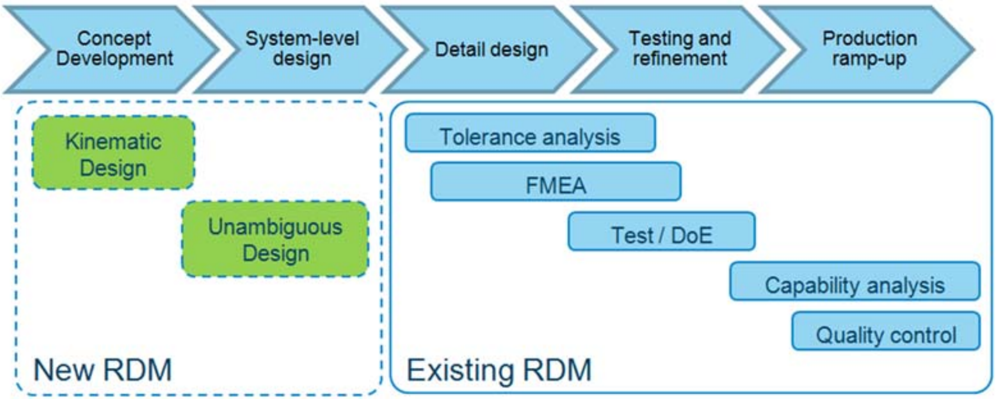


Figure 1. The generic product development process from Ulrich and Eppinger [Ulrich and Eppinger 1995] with Robust Design Methods placed according to the authors' experience of when they are typically applied in industry (In reality, the process is more iterative). The methods shown in the dashed boxes are new

In some robust design literature [e.g. Andersson 1996] it is pointed out that the existing Robust Design Methodologies are difficult – if not impossible – to apply to early-stage designs. Andersson argues, that current methods focus on analysis rather than synthesis, and that the methods are difficult for the typical design engineer to apply, due to their statistical and theoretical nature. Andersson continues through a list of design principles for the designer to use during early design stages. The principle of kinematic design is mentioned as a principle that results in robustness.

Pahl and Beitz [Pahl and Beitz 2007] provide a somewhat similar principle referred to as *design clarity*. However, it is not elaborated upon and developed into an operable tool for the design engineer to use. Downey [Downey 2003] describes a procedure for *smart assemblies,* which are also based on kinematically correct constraints.

Kinematic Design [Myszka 2005] is a design principle which focuses on obtaining a design which is not *overconstrained,* i.e. having more constraints than needed. Overconstrained designs entail a series of effects, some of which contribute to variation in functional performance. If a design is

overconstrained it will be more sensitive, with greater variations in functional performance caused by undesired variations of design parameters.
In Axiomatic Design [Suh 2005] there are two axioms –the independence axiom and the information axiom. The latter states, that *information* should be minimized in order to obtain robust designs, e.g. by reducing the number of design parameters that influence a given functional requirement. By nature, a kinematically overconstrained design is also a design with the potential for reducing the information content (in accordance with axiomatic design guidlines), since one or more constraints potentially can be removed.
It is seen, that there is a correlation between robust design and kinematic design in the sense that kinematic design is a means – among others – to obtain robust design. However, the principles are not elaborated upon and developed into systematic design tools to be used by the designer. The authors of this article, having worked with a wide array of product development projects in industry for more than a decade, have not yet seen kinematic design systematically applied in early phases of engineering design.
In some areas of engineering design, variation in functional performance is extremely important, e.g. measurement equipment and production equipment such as mills and lathes. Performance variation in these applications will result in increased measurement uncertainty and production tolerances, respectively. In robot and mechanism design, with many moving parts, overconstraints can result in jamming mechanisms, excessive loads (and hence product failures), noise and vibrations. Design guidelines for these types of products are actually based on kinematic principles and are called Exact Constraint Design, Minimum Constraint Design etc. One could argue, that it is expensive to apply principles from high-precision products on e.g. consumer products and that it would therefore only be viable to apply these principles to designs involving either high cost or extreme precision. However, kinematic design is merely a question of design principles, and as stated below, kinematic design principles can reduce tolerance requirements and hence production costs are reduced.
An aspect only rarely covered in literature is the aspect of ambiguity. Even though a design is kinematically correct designed in its nominal state, variation of the design parameters can change the interfaces and constraints of the design. For example, an extra constraint can be introduced this way, thereby reducing the mobility of the design. Alternatively, a constraint can switch from one surface to another, which obviously contributes to variation in functional performance. The aspect of ambiguity will be described in more detail later.
Concluding, current state-of-the-art contains many Robust Design Methods to be used for in-depth analyses of how the functional performance is affected by variations in the design parameters, but it lacks a simple, operable method for quantifying the clarity (or ambiguity) of the design. This means that there is a risk of sub-optimising the robustness of a design, which is conceptually sensitive, because it does not adher to the principles of Kinematic Design and Design Clarity.

3. Kinematic design at system level: Mobility

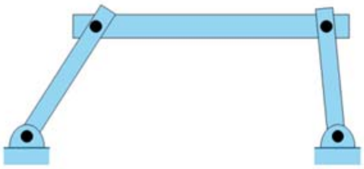
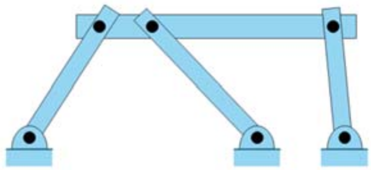
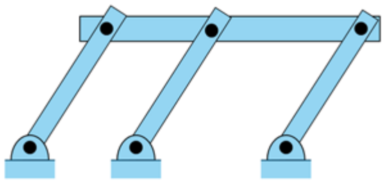
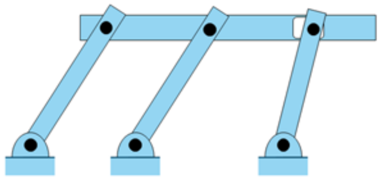
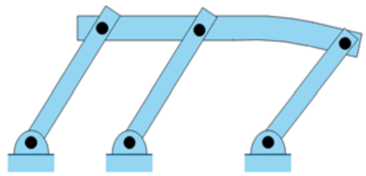
Kinematic design is normally used for designing mechanisms. For a mechanism, it is important that the system has the correct mobility. The mobility is calculated by using the Kutzbach-Gruebler formula, which uses the constraints and the number of elements in a system as inputs and results in a number describing the so-called mobility of the system, i.e. whether the system has the ideal number of constaints. Note that the formula can also be applied to static systems – the only difference being that the intended mobility is equal to 0.
The Kutzbach-Gruebler [Boe 1997] formula states:

$$M(3D) = 6(n-1) - \sum U - \sum F_{id} \qquad\qquad M(2D) = 3(n-1) - \sum U - \sum F_{id} \qquad (1)$$

Equation 1. The Kutzbach-Gruebler formula for a 3D and a 2D mechanism. M = system mobility, n = number of links/bodies, U = number of constraints, F_{id} = number of identical freedoms

For a mechanism, it is crucial that the system mobility according to the Kutzbach-Gruebler formula is correct, otherwise the mechanism may jam, experience noise, vibrations, wear, and/or excessive internal forces (also called parasitic loads), due to constraints ‘fighting against’ each other.

Table 1. Examples of applying the Kutzbach equation

<table>
<tr><td colspan="3">
Four-bar linkage mechanism with mobility = 3(4-1)-4*2=1, meaning that it has 1 degree of freedom, i.e. one input (such as a motor) will completely define all motions of the mechanism.</td><td colspan="2">
Five-bar mechanism with mobility = 3(5-1)-6*2 = 0, meaning it is fully constrained. If an input (such as a motor) is also applied, it will be overconstrained.</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td></td></tr>
<tr><td colspan="5">Mechanisms with mobility = 3(5-1)-6*2 = 0. Although the mechanisms are fully constrained, movement is still possible if
• Certain geometric requirements are fulfilled, the links with fixed ends must remain parallel at all times requiring tight toletances of lengths and joint positions (Left)
• Sufficient play is provided in the joint(s) (Middle)
• The link(s) have sufficient flexibility (Right)</td></tr>
</table>

In Table 1, an ideal kinematic linkage system is shown along with a series of designs that are overconstrained, if an input is also applied. It is seen how an overconstrained design must be compensated by one or more of the following:

- **Tolerances (Bottom left)**. In certain cases, an overconstrained mechanism can still be mobile, if it is produced with tight tolerances.
- **Clearance (Bottom middle)**. If the joints are designed with sufficient play, the mechanism can become mobile. At some point this clearance will transcend into an actual degree of freedom.
- **Flexibility (Bottom right)**. If the links in the mechanism are made of a flexible material, the mechanism can become mobile.

All of the above mitigative actions lead to an increase in functional performance variation or increased cost. Increased clearance in the bearings lead to greater variation in the position of the links, increased flexibility leads to higher deflections, when parts are exerted to loads, and tightened tolerances lead to increased production costs. It is important to mention, that many everyday designs are overconstrained, e.g. a ball bearing. In other words, designs may still function even though they are overconstrained, but they will always have to be compensated by one of the above principles (the balls for ball bearings are produced with extreme tolerances).

During early-stage design, often only a sketch of the design principle is available. However, kinematic design can easily be applied at this stage. To illustrate this, Figure 2 shows a principle for a windturbine with a shaft, a coupling a gearbox and a base (not shown). At this stage, it can be seen that this principle is overconstrained by 5, meaning that 5 constraints must be removed to obtain ideal mobility. This can be done in different ways, e.g. by replacing the current coupling which has 6 constraints, with one that only constrains the axial rotation (an Oldham-coupling, for example). Many alternative solutions can be synthesized in this way.

Body mobility											
Joint	ID	Name		X	Y	Z	Rx	Ry	Rz	F	U
1-2		Base	Shaft	0	0	1	0	0	1	2	4
2-3		Shaft	Coupling	0	0	0	0	0	0	0	6
3-4		Coupling	Gearbox	0	0	0	0	0	0	0	6
4-1		Gearbox	Base	0	0	0	0	0	0	0	6
N			4	Sum of constraints:							22
Fid			0								
No. of inputs			1								
B			-4	B(3D) = 6*(N-1)-Fid-Sum(u)							
Mobility			-5	M = B - Inputs							

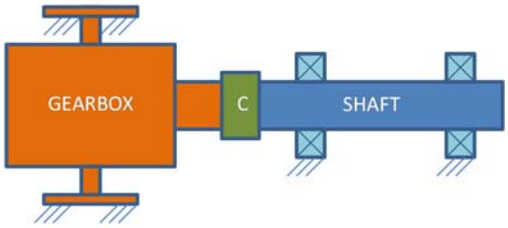


Figure 2. Kinematic analysis of a windturbine-design. The mobility is -5, meaning that 5 constraints have to be removed to obtain ideal mobility

By using the Kutzbach-Gruebler formula, the design engineer has a simple and fast tool to improve robustness in early-stage design. Efforts should be made to obtain a conceptual design where the mobility is as intended. Note that the Kutzbach-Gruebler formula does not need any design parameters or numeric values. Only a design sketch like the ones shown in Figure 2 are needed and hence it can be used at early design stages.

It should be noted, that the Kutzbach-Gruebler formula can also be used during the concept phase as a synthesis tool. By altering the number and type of constraints and links, a systematic array of concepts can be derived. As a final note, it is important to stress the difference between the intended constraints and the actual constraints. The procedure sketched above is merely a way to ensure that the intended constraints result in the desired mobility. Later in the design process, a review must be made to ensure compliance between the intended constraints and the actual constraints. This is done by using tolerance analysis, structural analysis etc. and is not covered in this paper.

Concluding, it has been shown that there is a correlation between the kinematic mobility and the robustness of a design. Thus, the mobility is a simple way to quantify the robustness of the design at an early-stage – the more overconstraints a design has, the more it will be prone to variations in functional performance.

4. Kinematic design at interface level: Design clarity

Once the system level architecture is defined, focus is shifted to the individual interfaces between the components in the product. Here, the Kutzbach-Gruebler equation can not be used in its pure form. However, the concept of ensuring that there are no superfluous constraints is still valid. All interfaces in the product should systematically be evaluated using a step-by-step process, thereby giving an overview of the interfaces that could be sensitive to variations.

Design Clarity Procedure

1. Identify interfaces, e.g. in an interface matrix. The result of this is an overview of all components that have functional surfaces against eachother.
2. Specify *intended* constraints for each interface. This can be done in a simple table like the one shown in Figure 3. For example the intended interface between a shaft and a journal bearing would be to have 1 free rotation (RZ) and the 5 remaining degrees of freedom (DOF) constrained.

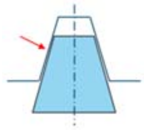
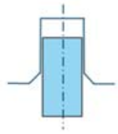
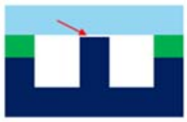
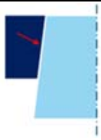
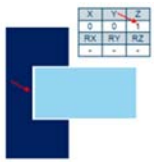
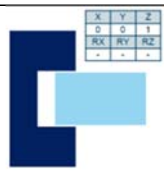
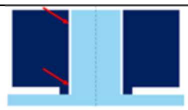
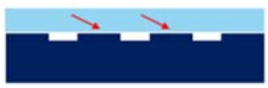
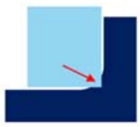
Intended Degrees of Freedom		
X	Y	Z
0	0	0
RX	RY	RZ
0	0	1
0 = constrained, 1 = free, -1 = overconstrained		

Actual Degrees of Freedom		
X	Y	Z
0	0	0
RX	RY	RZ
-1	-1	1
0 = constrained, 1 = free, -1 = overconstrained		

Figure 3. A description of the a) intended and b) actual constraints of a given interface. X,Y, and Z are the three translational DOFs. RX, RY, and RZ are the three rotational DOFs

3. For each interface, specify the *actual* clarity of each individual DOF using Table 2 as a reference. Each of the ambiguity principles that are not adhered to is regarded as a an extra constraint in the relevant DOF.

Table 2. Principles of clarity

ID	Type of ambiguity	Example	Solution
A	**Angled interface.** Coupling between angle, width and position of part.		
C	**Clearance < n*production_capability.** Risk of an unwanted constraint due to part variations.		
D	**Draft on interface element.** Draft angle defines positioning.		
F	**Flash.** Flash acts as interface element. Misplacement of component.		
I	**Intended not realized.** Loss of overview e.g. wrt. tolerance and structural analysis		
L	**Large surface.** Increases demand on form tolerances.		
M	**Multiple surfaces constrain same DOF.** Loss of overview. Parasitic loads. Increase in tolerance demands.		
R	**Round.** Round acts as interface element. Misplacement of component.		
S	**Shift (abrupt) of interface.** Sudden change of contact point		

4. List functional performance requirements and the design parameters that contribute to the functional performance – change design to make parameters obsolete
5. Commence using traditional Robust Design Methodologies to optimize remaining parameters.

The consequences of not having clarity in the design are:

- **Abrupt functional changes**. An abrupt functional change is defined as a change of function due to an infinitisemally small variation of a design parameter.
- **Variation in functional performance,** e.g. component placement, required assembly forces, etc.
- **Reduced precision in tolerance analysis.** If the constraints are ambiguous, tolerance chains will also be ambiguous.
- **Reduced precision in structural analysis.** If the constraints are ambiguous, exact constraints for the structural analysis cannot be defined.

An overview of the constraints in the entire system can be valuable to e.g. the project mangager, or for a project review team, wanting to ensure that all interfaces have been reviewed. An example of this is shown in Figure 4, where each row constitutes an interface, and is followed by the intended and actual DOFs. Any discrepancy between the two results can be visualised by e.g. coloring. The current ambiguity of the design can be summed up simply by adding the number of ambiguities in each of the interfaces.

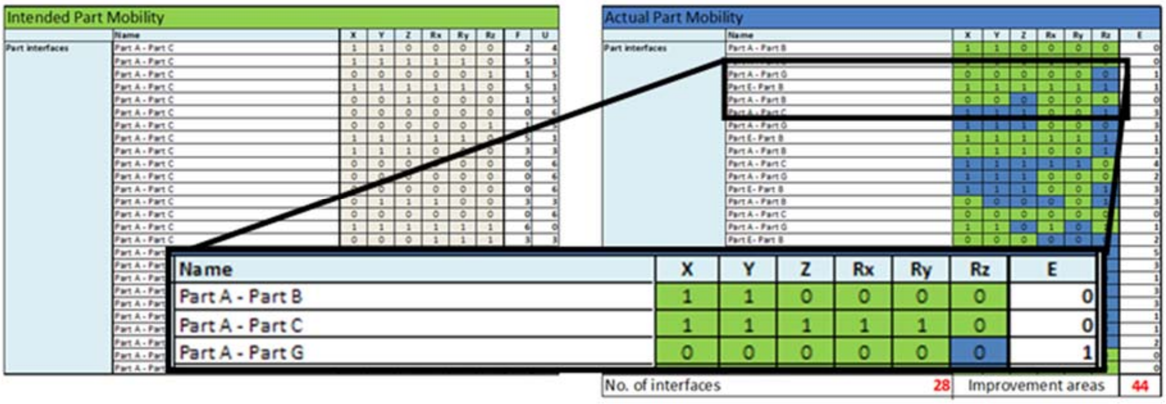


Figure 4. Overview of all interfaces in the product. Each row contains an interface, The left table contains the intended DOF. The right contains the actual DOF. If the actual DOF is different from the intended DOF, the cell is colored to mark that there is a potential robust design improvement. In the bottom row, the overall ambiguity is quantified

Concluding, the method is fast and simple to use for the design engineer, and hence it is possible to use it during synthesis of the detailed design. It has been shown how the design clarity and robustness are correlated and how the design clarity can be quantified.

5. Kinematic design and design clarity – primers for robust design

The effect of quantifying and optimising the mobility and the clarity of the design will be a reduced variation in functional performance due to the combination of a more precise identification of the design's functional surfaces and influencing design paramaters and the following reduction of the number of design parameters. In other words, a design with an ideal mobility and an ideal set of constraints, will perform in a more consistent and predictable manner than a similar design which is overconstained and ambiguous.

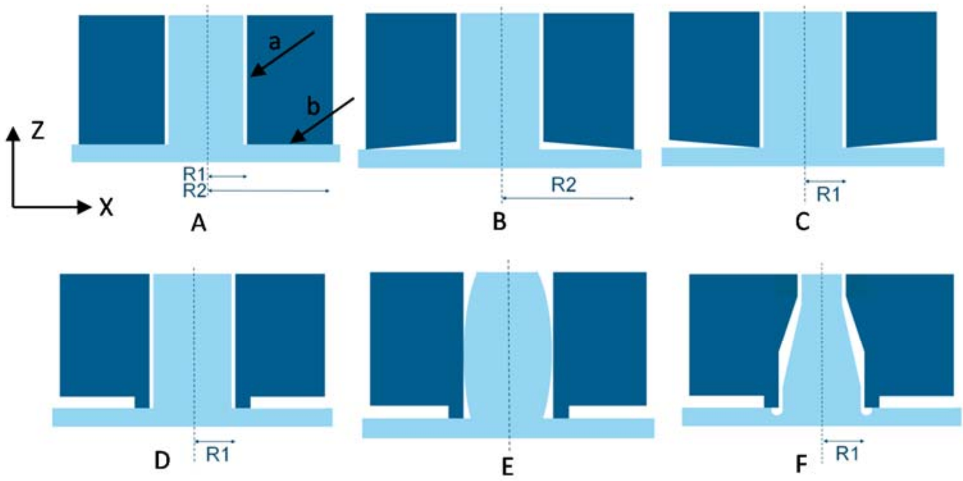
The subsequent task of quantifying and optimising the robustness of the design using traditional Robust Design Methods, will naturally be simpler to carry out on a design with ideal constraints and low ambiguity. Furthermore, testing and ramp-up of the design will become simpler due to the improved repeatability of the design. This can help reduce the development time and increase the precision of the project execution.

6. Cases

Case 1: Gearwheel

The case is based on a gearwheel which revolves around a center pin – see figure 5. The step-by-step procedure is used. **Step 1** is elementary, since there are only 2 parts. In **Step 2**, it is seen that the intent of the design is to constrain all degrees of freedom except the axial rotation RZ. This is added to

Step 3: Using the principles in Table 2, it is seen that the design in Figure 5A contains several ambiguities. The principles F,M,L,R,S are violated, meaning that the design is ambiguous. In Figure 5-a it is seen that interface element 'a' constrains the X- and Y-directions. Element 'b' constrains the Z-direction. However, both a and b constrain RX and RY (M-principle). The round and flash at the bottom of the shaft can affect component placement (F,R). An infinitely small change of the angle of the bottom surface of the gearwheel can abruptly change the contact point between the gearwheel and the shaft (Figures 5b/c), hence changing the friction significantly (S).



Intended Degrees of Freedom		
X	Y	Z
0	0	0
RX	RY	RZ
0	0	1
0 = constrained, 1 = free, -1 = overconstrained		

Actual Degrees of Freedom		
X	Y	Z
-3	-3	-3
RX	RY	RZ
-1	-1	1
0 = constrained, 1 = free, -1 = overconstrained		

Figure 5. A gearwheel (dark part) runs on a center pin. The objective is to have low friction and a minimum of wobbling

The large contact surface between the shaft and the gearwheel puts demands on form tolerances (L), otherwise the gearwheel may 'wobble' during operation (Figure 5d), thereby creating noise. In Figure 5e, a design is proposed, which is very close to ideal Design Clarity. The Design Ambiguity has gone from 11 to 0.

Step 4 – List functional requirements.

1. Minimum rotational friction (because friction reduces gear efficiency)
2. Minimum wobbling around X- & Y-axes. Wobbling contributes to noise from the gear.

Step 5 – List design parameters.

Now, it is more clear which design parameters influence the functional performance and hence, traditional Robust Design Methods can now be introduced, optimising the parameters of the design. Also, tolerance and structural analyses will benefit from the improved design clarity.

Case 2: Pin Assembly

This case is based on an interface often seen in industrial applications – see Figure 6.

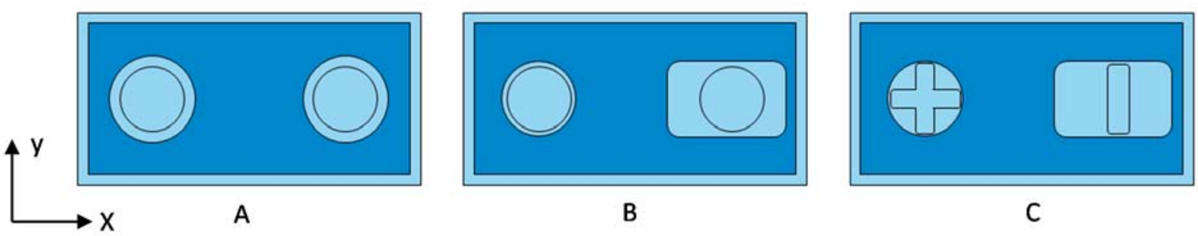


Figure 6. Interface between a component with two pins and a component with two holes. All 6 DOFs are to be constrained. In A) the design is ambiguous. In B) the design is improved. In C) the design has been further optimised for robustness, thereby reducing the variance of the holding force

Using the step-by-step procedure:
Step 1 – Identify interfaces. There are only two parts, and they interface with each other.
Step 2 – Intended DOFs. The two components should act as a single body, i.e. all 6 DOFs are intended to be constrained.

Intended Degrees of Freedom		
X	Y	Z
0	0	0
RX	RY	RZ
0	0	0
0 = constrained, 1 = free, -1 = overconstrained		

Actual Degrees of Freedom		
X	Y	Z
-1	-1	0
RX	RY	RZ
0	0	0
0 = constrained, 1 = free, -1 = overconstrained		

Step 3 – Principles of clarity
In Figure 6-a it is seen that both pins control the X- and Y-directions (M-principle). The remaining constraints are as intended (it can be argued that RX and RY are overconstrained but here the length of the interface between the pins and the holes is assumed to be so short that the interface does not control RX and RY. In Figure 6-b an alternative design is suggested, with the left pin controlling X and Y and the right pin controlling only RZ. Using the same tolerances, it is now possible to create a closer fit between the pins and holes.
Step 4 – List functional requirements.

1. Position tolerance of component placement
2. Stress level in components lower then tensile stress

Step 5 – List design parameters.
Having reduced the size of the interface elements of the pressfit, it will be possible to design with a larger overlap between the two parts, without exceeding the allowable stress levels of the materials. Due to the larger overlap, the influence of the tolerances wrt. the holding force will be redueced and thus the design is more robust.

7. Conclusions

In this paper, a review of literature has shown that there is a lack of specific and operational methods and tools for early-stage synthesis of robust designs. However, it is also shown that Kinematic Designs and designs with high Design Clarity are more robust against variations in design parameters than a corresponding design which is overconstrained and ambiguous.
During concept design, it is suggested to use the Kutzbach-Grueblerformula to secure that the mobility of the concept is as intended, as overconstrained designs can lead to parasitic loads and variation in product lifetime, noise, vibrations and unwanted deflections.
During the interface design phase, a step-by-step method is proposed for systematically analysing all interfaces and identifying any ambiguities. This is done using the specific set of Clarity Principles. Failure to remove ambiguities in the design can lead to component misplacements, lack of precision in tolerance and structural analyses and abrupt functional changes, which again can result in variation in functional performance.
When attempts have been made to obtain a kinematically correct and unambiguous design, traditional Robust Design Methods can be implemented, focusing on further optimising the design parameters wrt. robustness.

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