INTERNATIONAL DESIGN CONFERENCE - DESIGN 2004
Dubrovnik, May 18 - 21, 2004.

DESIGN 2004

INFLUENCE OF PITCH ERRORS ON LOAD DISTRIBUTION ON SPUR INVOLUTE HCR GEARS TEETH

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Keywords: pitch error, load distribution, high contact ratio gears

1. Introduction

High contact ratio gears (HCR) are more frequently in use because of their increased load-carrying capacity, compared to low contact ratio gears [Lovrin 2001], although they are very sensitive to manufacturing errors. The choice of quality grade has the major influence on load distribution on mating teeth, and consequently on increase or decrease of tooth root and contact stresses. In this paper the expressions for calculation of load distribution on spur involute HCR gears teeth with pitch errors are established.

2. Load distribution on high contact ratio gears

High contact ratio gears have transverse contact ratio higher than two. They have two or three tooth pairs in contact during the mesh. Triple contact exists from point A to B, D to E and F to G on the pressure line, while double contact exists from point B to D and from E to F (Fig. 1). Hypothetically, the tooth pair is transmitting 33,33% of the whole load in triple contact and 50% of the whole load in double contact. A theoretical load distribution exists when gears are geometrically ideal and the teeth are elastically deforming. The value of the pitch error is defined by the choice of quality grade. When increasing the pitch error, which means choosing coarser quality grade, load distribution increasingly deviates from theoretical values.

3. Force distribution factors

The whole load applied on the gear is sum of load shares on tooth pairs in triple and double contact:

$$F_{\mathrm{bt}} = F_{\mathrm{bti(A\text{-}B)}} + F_{\mathrm{bti(D\text{-}E)}} + F_{\mathrm{bti(F\text{-}G)}} \tag{1}$$

$$F_{\mathrm{bt}} = F_{\mathrm{bti(B\text{-}D)}} + F_{\mathrm{bti(E\text{-}F)}} \tag{2}$$

The force distribution factor for one tooth pair is ratio of the load share on this tooth pair and the whole load applied on the gear. It is defined on the basis of equality of elastic deformation of the tooth pairs in simultaneous contact [Petersen 1989] for geometrically ideal gears:

$$\frac{F_{\text{bti(A-B)}}}{c'_{\text{i(A-B)}}\cdot b}=\frac{F_{\text{bti(D-E)}}}{c'_{\text{i(D-E)}}\cdot b}=\frac{F_{\text{bti(F-G)}}}{c'_{\text{i(F-G)}}\cdot b} \tag{3}$$

$$\frac{F_{\text{bti(B-D)}}}{c'_{\text{i(B-D)}}\cdot b}=\frac{F_{\text{bti(E-F)}}}{c'_{\text{i(E-F)}}\cdot b} \tag{4}$$

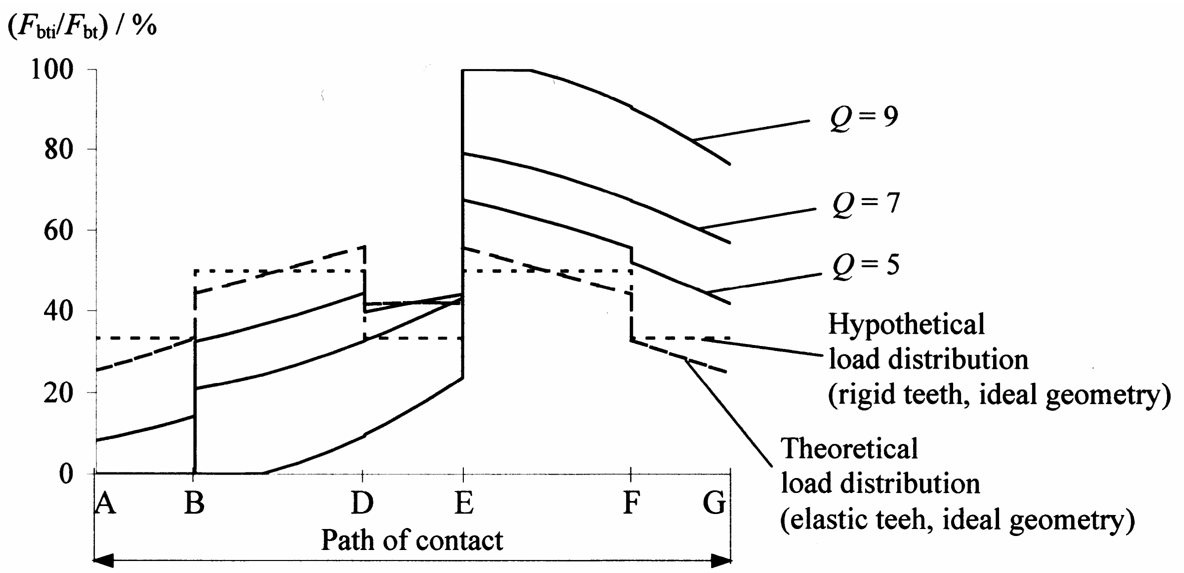

Figure 1. Load distribution on HCR gears

Force distribution factors for triple-double contact are:

$$\beta_{\text{0i(A-B)}}=\frac{F_{\text{bti(A-B)}}}{F_{\text{bt}}}=\left(1+\frac{c'_{\text{i(D-E)}}}{c'_{\text{i(A-B)}}}+\frac{c'_{\text{i(F-G)}}}{c'_{\text{i(A-B)}}}\right)^{-1} \tag{5}$$

$$\beta_{\text{0i(D-E)}}=\frac{F_{\text{bti(D-E)}}}{F_{\text{bt}}}=\left(1+\frac{c'_{\text{i(A-B)}}}{c'_{\text{i(D-E)}}}+\frac{c'_{\text{i(F-G)}}}{c'_{\text{i(D-E)}}}\right)^{-1} \tag{6}$$

$$\beta_{\text{0i(B-D)}}=\frac{F_{\text{bti(B-D)}}}{F_{\text{bt}}}=\left(\frac{c'_{\text{i(E-F)}}}{c'_{\text{i(B-D)}}}+1\right)^{-1} \tag{7}$$

The force distribution factor depends on the stiffness of the tooth pair. When the gears are in mesh, the mating teeth are elastically deforming. Stiffness of the teeth c'_i can be calculated by using the teeth elastic deformation δ_i, provided by Terauchi-Nagamura expressions [Terauchi, Nagamura 1981].

4. Load shares on mating teeth

When the gears are rotating, the values of pitch errors are changing from one tooth to another. Therefore the calculation of load shares should be performed when the gears in mesh are in the critical positions, which are shown on figures 2 and 3.

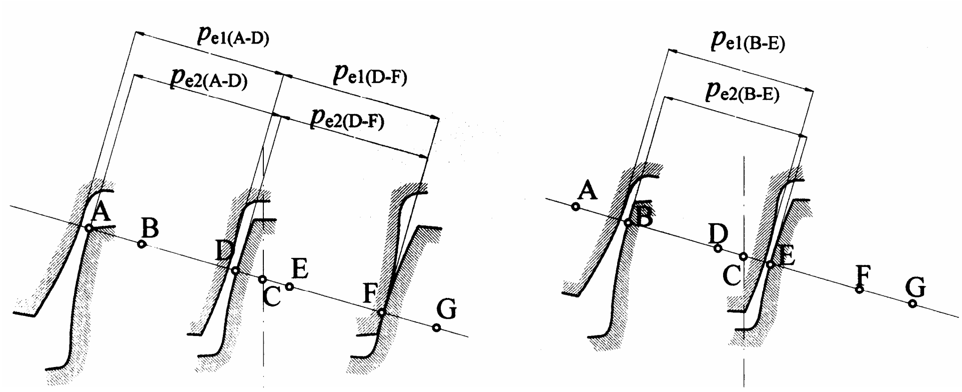

Figure 2. Critical position of pitch errors in triple and double contact with

$p_{e1(D\text{-}F)} > p_{e2(D\text{-}F)}$, $p_{e1(A\text{-}D)} + p_{e1(D\text{-}F)} > p_{e2(A\text{-}D)} + p_{e2(D\text{-}F)}$

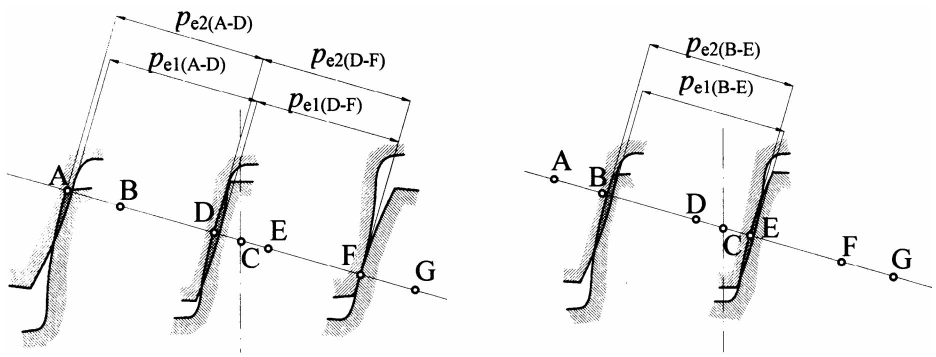

Figure 3. Critical position of pitch errors in triple and double contact with

$p_{e1(D\text{-}F)} < p_{e2(D\text{-}F)}$, $p_{e1(A\text{-}D)} + p_{e1(D\text{-}F)} < p_{e2(A\text{-}D)} + p_{e2(D\text{-}F)}$

The pitch errors for these positions of gears are:

$$f_{pe}^{'} = p_{e2(A\text{-}D)} + p_{e2(D\text{-}F)} - p_{e1(A\text{-}D)} - p_{e1(D\text{-}F)} \tag{8}$$

$$f_{pe}^{''} = p_{e2(D\text{-}F)} - p_{e1(D\text{-}F)} \tag{9}$$

The conditions of equal elastic deformation for gears with pitch errors are mathematically expressed:

$$\delta_{i(A\text{-}B)} = \frac{F_{bti(A\text{-}B)}}{c_{i(A\text{-}B)}^{'} \cdot b} - f_{pe}^{'} = \delta_{i(D\text{-}E)} = \frac{F_{bti(D\text{-}E)}}{c_{i(D\text{-}E)}^{'} \cdot b} - f_{pe}^{''} = \delta_{i(F\text{-}G)} = \frac{F_{bti(F\text{-}G)}}{c_{i(F\text{-}G)}^{'} \cdot b} \tag{10}$$

$$\delta_{i(B\text{-}D)} = \frac{F_{bti(B\text{-}D)}}{c_{i(B\text{-}D)}^{'} \cdot b} - f_{pe}^{'} = \delta_{i(E\text{-}F)} = \frac{F_{bti(E\text{-}F)}}{c_{i(E\text{-}F)}^{'} \cdot b} - f_{pe}^{''} \tag{11}$$

The expressions for calculation of load shares on mating teeth are developed by using the force distribution factors and the values of pitch errors, from equations (1), (2), (10) and (11).
Load shares on tooth pairs in triple contact are:

$$F_{\text{bti(A-B)}} = \beta_{0\text{i(A-B)}} \cdot \left[F_{\text{bt}} + f'_{\text{pe}} \cdot b \cdot \left(c'_{\text{i(D-E)}} + c'_{\text{i(F-G)}}\right) - f''_{\text{pe}} \cdot c'_{\text{i(D-E)}} \cdot b\right] \quad (12)$$

$$F_{\text{bti(D-E)}} = \beta_{0\text{i(D-E)}} \cdot \left[F_{\text{bt}} + f''_{\text{pe}} \cdot b \cdot \left(c'_{\text{i(A-B)}} + c'_{\text{i(F-G)}}\right) - f'_{\text{pe}} \cdot c'_{\text{i(A-B)}} \cdot b\right] \quad (13)$$

$$F_{\text{bti(F-G)}} = F_{\text{bt}} - F_{\text{bti(A-B)}} - F_{\text{bti(D-E)}} \quad (14)$$

Load shares on tooth pairs in double contact are:

$$F_{\text{bti(B-D)}} = \beta_{0\text{i(B-D)}} \cdot \left[F_{\text{bt}} + c'_{\text{i(E-F)}} \cdot b \cdot \left(f'_{\text{pe}} + f''_{\text{pe}}\right)\right] \quad (15)$$

$$F_{\text{bti(E-F)}} = F_{\text{bt}} - F_{\text{bti(B-D)}} \quad (16)$$

5. Control and correction

If the pitch error is big (coarse quality grade), it is possible that in theoretically triple contact only two tooth pairs or even one tooth pair are in real contact, or in theoretically double contact only one tooth pair is in real contact. This occurs mostly when small forces are applied. The existence of the contact between two teeth can be controlled by the values of calculated load shares from the expressions (12) till (16), when presuming triple-double contact. The conditions for the contact existence are established (Table 1).
If the number of tooth pairs in simultaneous contact is changed, new conditions of equal elastic deformations of the tooth pairs are posed (Table 2) and corrected force distribution factors are developed:

$$\beta^*_{0\text{i(A-B)}} = \left(\frac{c'_{\text{i(D-E)}}}{c'_{\text{i(A-B)}}} + 1\right)^{-1} \quad (17)$$

$$\beta^*_{0\text{i(D-E)}} = \left(\frac{c'_{\text{i(F-G)}}}{c'_{\text{i(D-E)}}} + 1\right)^{-1} \quad (18)$$

$$\beta^*_{0\text{i(F-G)}} = \left(\frac{c'_{\text{i(A-B)}}}{c'_{\text{i(F-G)}}} + 1\right)^{-1} \quad (19)$$

Table 1. Conditions for the contact existence

Tooth pairs in contact	Conditions for the contact existence
A-B and D-E, but not F-G	$F_{\text{bti(A-B)}} + F_{\text{bti(D-E)}} > F_{\text{bt}}$, $F_{\text{bti(A-B)}} < F_{\text{bt}}$, $F_{\text{bti(D-E)}} < F_{\text{bt}}$
D-E and F-G, but not A-B	$F_{\text{bti(D-E)}} + F_{\text{bti(F-G)}} > F_{\text{bt}}$, $F_{\text{bti(D-E)}} < F_{\text{bt}}$, $F_{\text{bti(F-G)}} < F_{\text{bt}}$
A-B and F-G, but not D-E	$F_{\text{bti(A-B)}} + F_{\text{bti(F-G)}} > F_{\text{bt}}$, $F_{\text{bti(A-B)}} < F_{\text{bt}}$, $F_{\text{bti(F-G)}} < F_{\text{bt}}$
A-B, but not D-E and F-G	$F_{\text{bti(A-B)}} > F_{\text{bt}}$, $F_{\text{bti(A-B)}} > F_{\text{bti(D-E)}}$, $F_{\text{bti(A-B)}} > F_{\text{bti(F-G)}}$
B-D, but not E-F	$F_{\text{bti(B-D)}} > F_{\text{bt}}$
D-E, but not A-B and F-G	$F_{\text{bti(D-E)}} > F_{\text{bt}}$, $F_{\text{bti(D-E)}} > F_{\text{bti(A-B)}}$, $F_{\text{bti(D-E)}} > F_{\text{bti(F-G)}}$
E-F, but not B-D	$F_{\text{bti(E-F)}} > F_{\text{bt}}$
F-G, but not A-B and D-E	$F_{\text{bti(F-G)}} > F_{\text{bt}}$, $F_{\text{bti(F-G)}} > F_{\text{bti(A-B)}}$, $F_{\text{bti(F-G)}} > F_{\text{bti(D-E)}}$

Table 2. Conditions of equal elastic deformations

Tooth pairs in contact	Conditions of equal elastic deformations
A-B and D-E, but not F-G	$\delta_{\text{i(A-B)}} = \dfrac{F_{\text{bti(A-B)}}}{c'_{\text{i(A-B)}} \cdot b} - f'_{\text{pe}} = \delta_{\text{i(D-E)}} = \dfrac{F_{\text{bti(D-E)}}}{c'_{\text{i(D-E)}} \cdot b} - f''_{\text{pe}}$
D-E and F-G, but not A-B	$\delta_{\text{i(D-E)}} = \dfrac{F_{\text{bti(D-E)}}}{c'_{\text{i(D-E)}} \cdot b} - f''_{\text{pe}} = \delta_{\text{i(F-G)}} = \dfrac{F_{\text{bti(F-G)}}}{c'_{\text{i(F-G)}} \cdot b}$
A-B and F-G, but not D-E	$\delta_{\text{i(A-B)}} = \dfrac{F_{\text{bti(A-B)}}}{c'_{\text{i(A-B)}} \cdot b} - f_{\text{pe}} = \delta_{\text{i(F-G)}} = \dfrac{F_{\text{bti(F-G)}}}{c'_{\text{i(F-G)}} \cdot b}$

Corrected expressions for calculation of load share are defined (Table 3).

Table 3. Corrected load shares

Tooth pairs in contact	Load shares
A-B and D-E, but not F-G	$F_{\text{bti(A-B)}} = \beta^*_{\text{0i(A-B)}} \cdot \left[F_{\text{bt}} + c'_{\text{i(D-E)}} \cdot b \cdot \left(f'_{\text{pe}} - f''_{\text{pe}}\right)\right]$, $F_{\text{bti(D-E)}} = F_{\text{bt}} - F_{\text{bti(A-B)}}$
D-E and F-G, but not A-B	$F_{\text{bti(D-E)}} = \beta^*_{\text{0i(D-E)}} \cdot \left(F_{\text{bt}} + f''_{\text{pe}} \cdot c'_{\text{i(F-G)}} \cdot b\right)$, $F_{\text{bti(F-G)}} = F_{\text{bt}} - F_{\text{bti(D-E)}}$
A-B and F-G, but not D-E	$F_{\text{bti(F-G)}} = \beta^*_{\text{0i(F-G)}} \cdot \left(F_{\text{bt}} - f'_{\text{pe}} \cdot c'_{\text{i(A-B)}} \cdot b\right)$,

	$F_{\text{bti(A-B)}} = F_{\text{bt}} - F_{\text{bti(F-G)}}$
A-B, but not D-E and F-G	$F_{\text{bti(A-B)}} = F_{\text{bt}}$
B-D, but not E-F	$F_{\text{bti(B-D)}} = F_{\text{bt}}$
D-E, but not A-B and F-G	$F_{\text{bti(D-E)}} = F_{\text{bt}}$
E-F, but not B-D	$F_{\text{bti(E-F)}} = F_{\text{bt}}$
F-G, but not A-B and D-E	$F_{\text{bti(F-G)}} = F_{\text{bt}}$

6. Conclusion

Pitch errors have major influence on load distribution on mating teeth and consequently on tooth root and contact stresses. The stresses can be significantly increased if theoretical contact isn't equal to the real contact. The expressions for the calculation of load shares are provided for the real mesh. The use of these expressions enables better estimation of required gear quality grade and more precise calculation of load-carrying capacity of gears.

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